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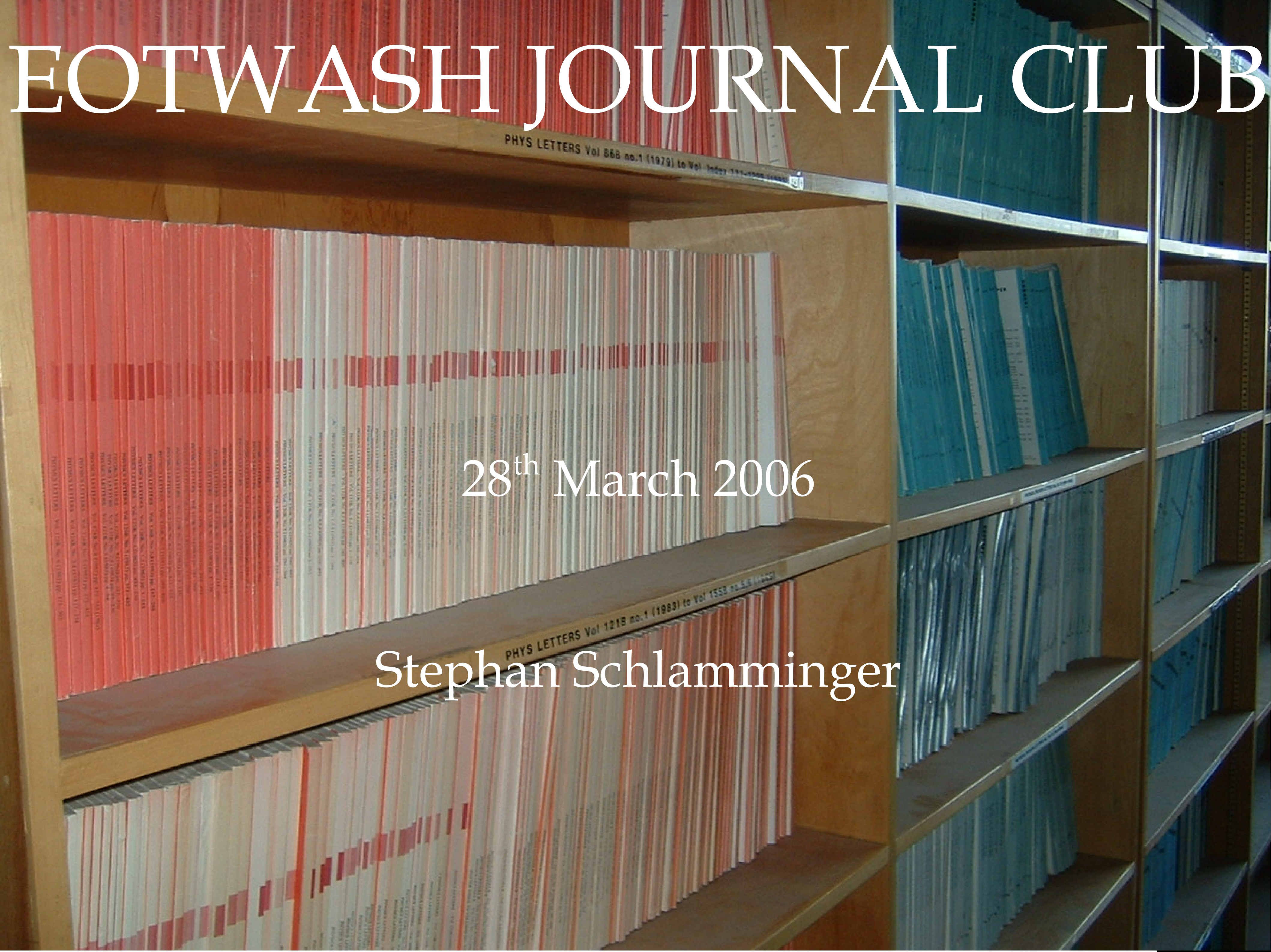

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Thermal Noise in mechanical experiments
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Thermal noise in mechanical experiments

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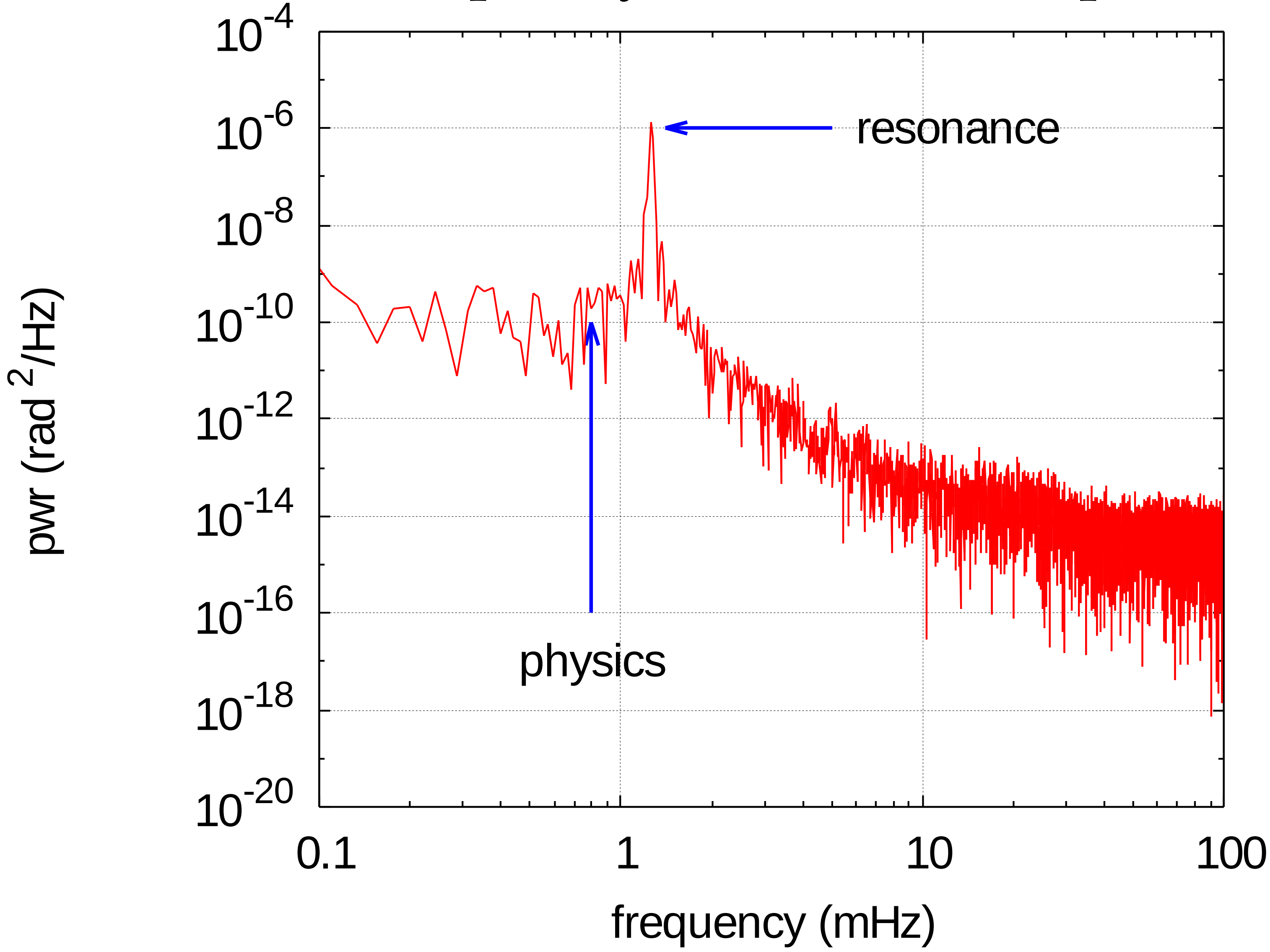
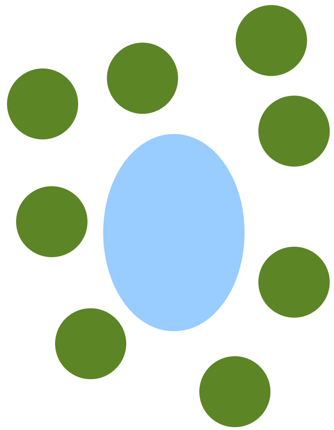
The fluctuation-dissipation theorem is applied to the case of low-dissipation mechanical oscillators, whose losses are dominated by processes occurring inside the material of which the oscillators are made. In the common case of losses described by a complex spring constant with a constant imaginary part, the thermal noise displacement power spectrum is steeper by one power of ω than is predicted by a velocity-damping model. I construct models for the thermal noise spectra of systems with more than one mode of vibration, and evaluate a model of a specific design of pendulum suspension for the test masses in a gravitational-wave interferometer.

"In many experiments, it is the thermal noise far from the resonant frequency that is most important"



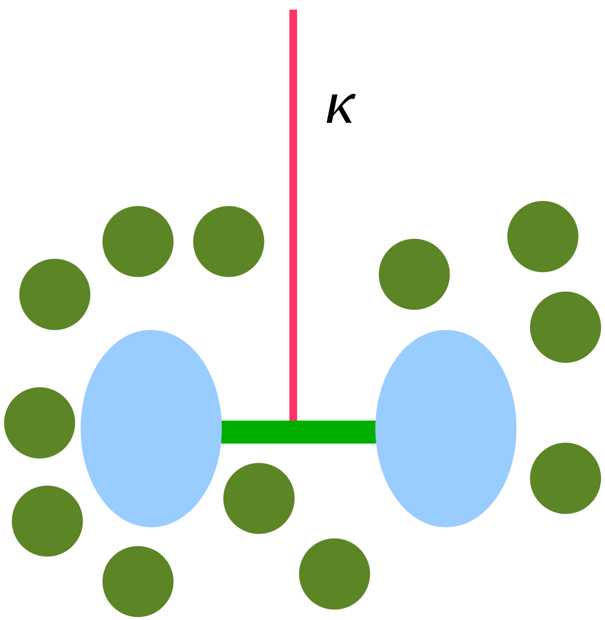
Brownian Motion



Langevin equation: $m\ddot{x} + f\,\dot{x} = F_{th}$

with $F_{th}^2(\omega) = 4\,k_B T\, f$ (white)

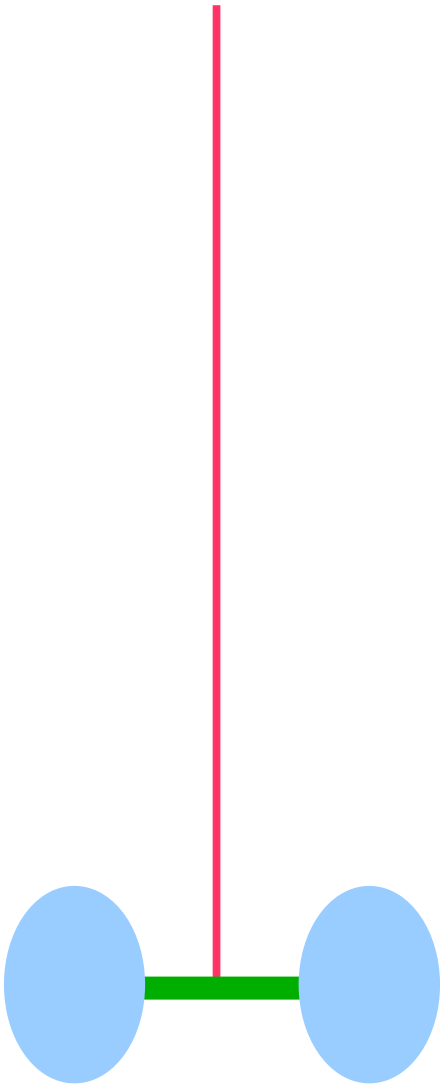
solution: $\langle \dot{x} \rangle = 0,\qquad \langle \dot{x}^2 \rangle = \dfrac{k_B T}{m}$



For a oscillator: $I\,\ddot{\varphi} + f\,\dot{\varphi} + \kappa\,\varphi = \tau_{th}$

solution: $$\varphi^2(\omega) = \frac{4\mathrm{k}_B T f}{(\kappa - I\,\omega^2)^2 + f^2\,\omega^2}$$

Fluctuation-Dissipation Theorem



$$\tau_{th}^2(\omega) = 4 k_B T R(\omega)$$

$$R(\omega) = \Re(Z(\omega))$$

mechanical resistance = real part of the impedance Z(w)

$$Z(\omega) = \frac{\tau}{v}$$

For us, Z is a very unusual quantity. We are used to think in **response function r.**

$$r(\omega) = \frac{\varphi(\omega)}{\tau(\omega)} \longrightarrow Z(\omega) = \frac{1}{i\omega r}$$

$$I\ddot{\varphi}+f\dot{\varphi}+\kappa\varphi=\tau(\omega)$$

$$\tau(\omega)=A\exp(i\omega t) \qquad \varphi=B\exp(i\omega t) \qquad \dot{\varphi}=i\omega B\exp(i\omega t)$$

$$-\omega^2 I B+f i B\omega+\kappa B=A \qquad Z(\omega)=\frac{\tau}{v}=\frac{A}{i\omega B}$$

$$\frac{A}{i\omega B}=i I\omega+f+\frac{\kappa}{i\omega} \quad \rightarrow R=f \quad \rightarrow F_{th}^2=4\mathrm{k}_B T f$$

Response Fkt.: $r=\dfrac{\varphi(\omega)}{\tau(\omega)}=\dfrac{B}{A}=\dfrac{1}{-\omega^2 I+i f\omega+\kappa}=\dfrac{1}{i\omega Z}$

$$\varphi(\omega)^2=\frac{\tau}{-\omega^2 I+i f\omega+\kappa}\,\frac{\tau}{-\omega^2 I-i f\omega+\kappa}=\frac{\tau^2}{(\kappa-I\omega^2)^2+\omega^2 f^2}$$

$$\varphi(\omega)^2=\frac{4k_B T}{I^2(\omega_0^2-\omega^2)^2+\omega^2 f^2}$$

External velocity damping

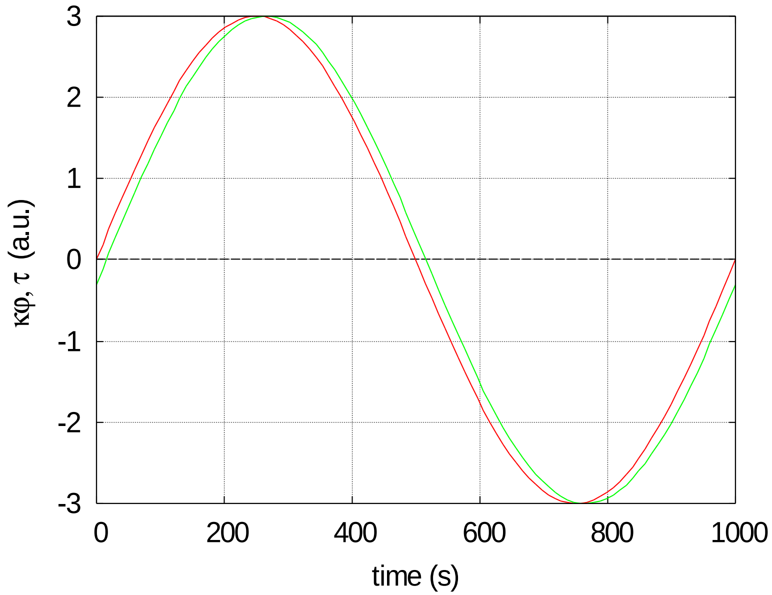
- Viscous gas damping
  - $Q_{gas} = C\,h \frac{\rho\,\omega_0 \sqrt{k_B T}}{p\sqrt{m}}$ Newwash: Q~3x10^7
- Eddy current damping
  - Good magnetic shielding, use of nonconductors
- In a good experimental design all sources of external damping can be eliminated!

Internal Damping

$$\tau = -\kappa(1 + i\,\delta(\omega))\,\phi$$

Loss angle as a function of frequency

$$I\,\ddot{\varphi} = -\kappa(1 + i\,\delta)\,\varphi + \tau$$

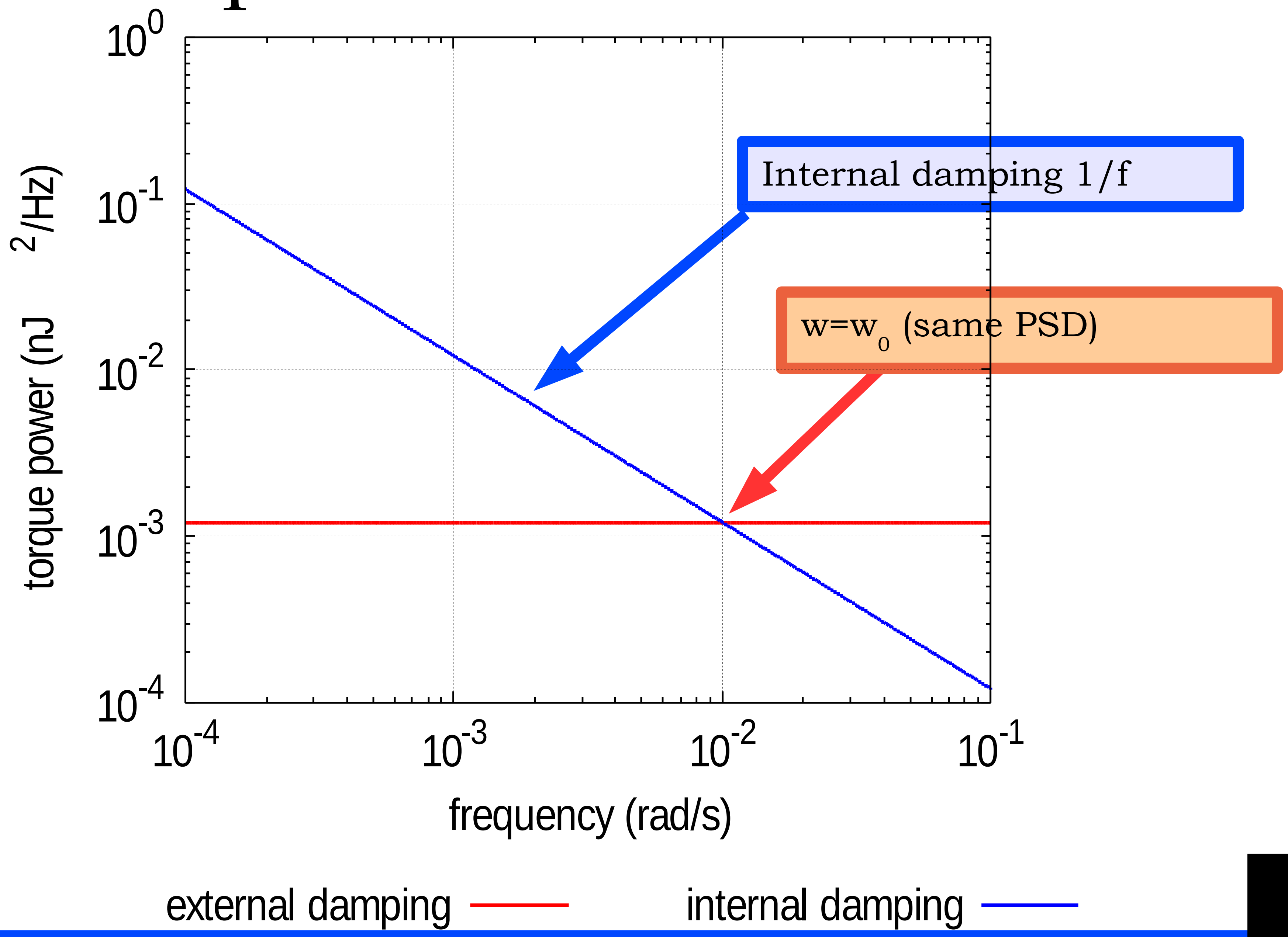


$$r = \frac{\varphi(\omega)}{\tau(\omega)} = \frac{1}{\kappa}\,\frac{1 - \frac{\omega^2}{\omega_0^2} - i\,\delta}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \delta^2}$$

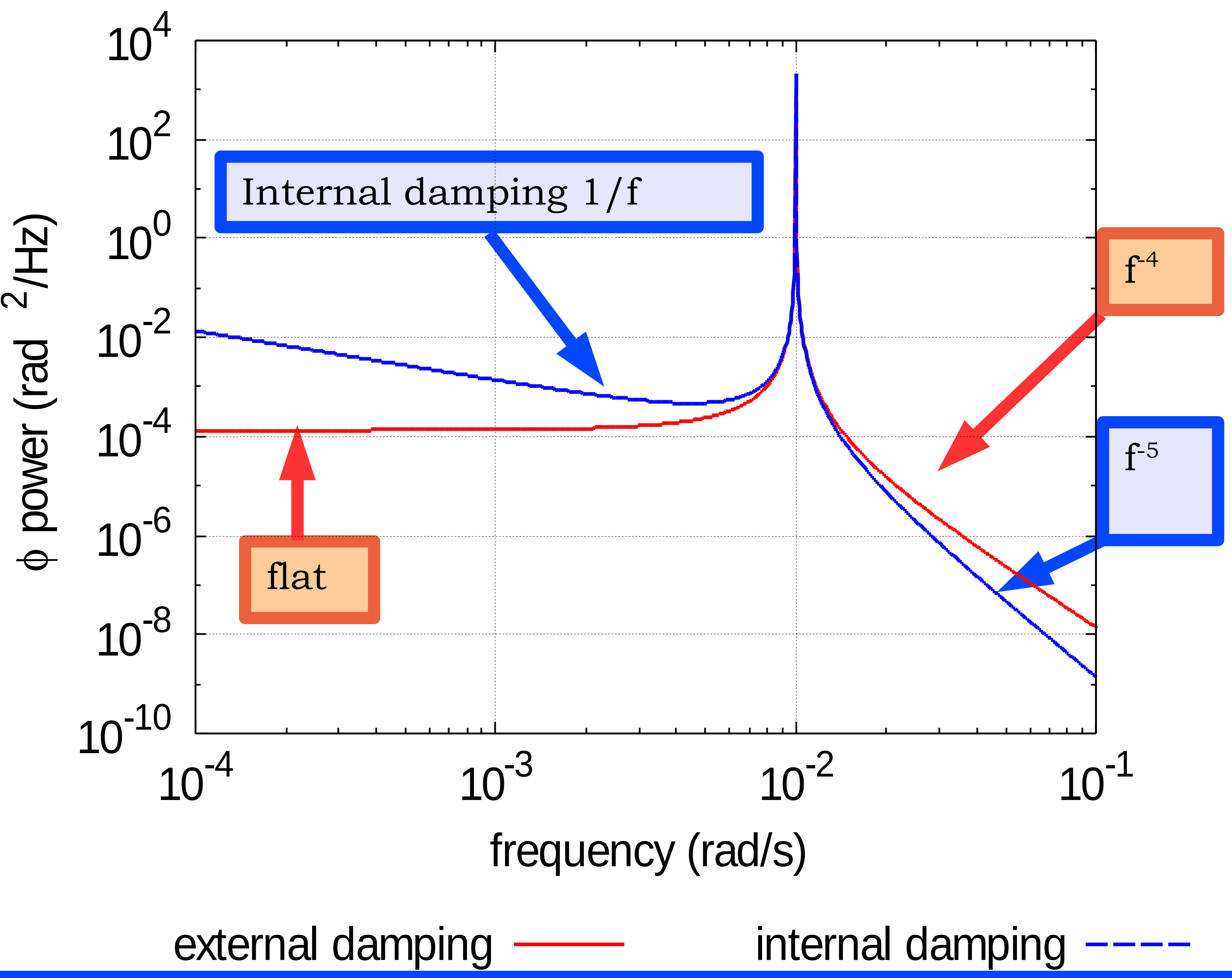
$$Z = \frac{1}{i\,\omega\,r} = \frac{-\kappa}{\omega}\left(i - \delta - i\,\frac{\omega^2}{\omega_0^2}\right) \Rightarrow \tau^2(\omega) = 4k_B T\,\frac{\kappa\,\delta}{\omega}$$

$$\varphi^2(\omega) = \frac{4\,k_B T\,\delta}{\omega\,\kappa\left(\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \delta^2\right)}$$

Comparison of the Noise PSD



Comparison of the position PSD

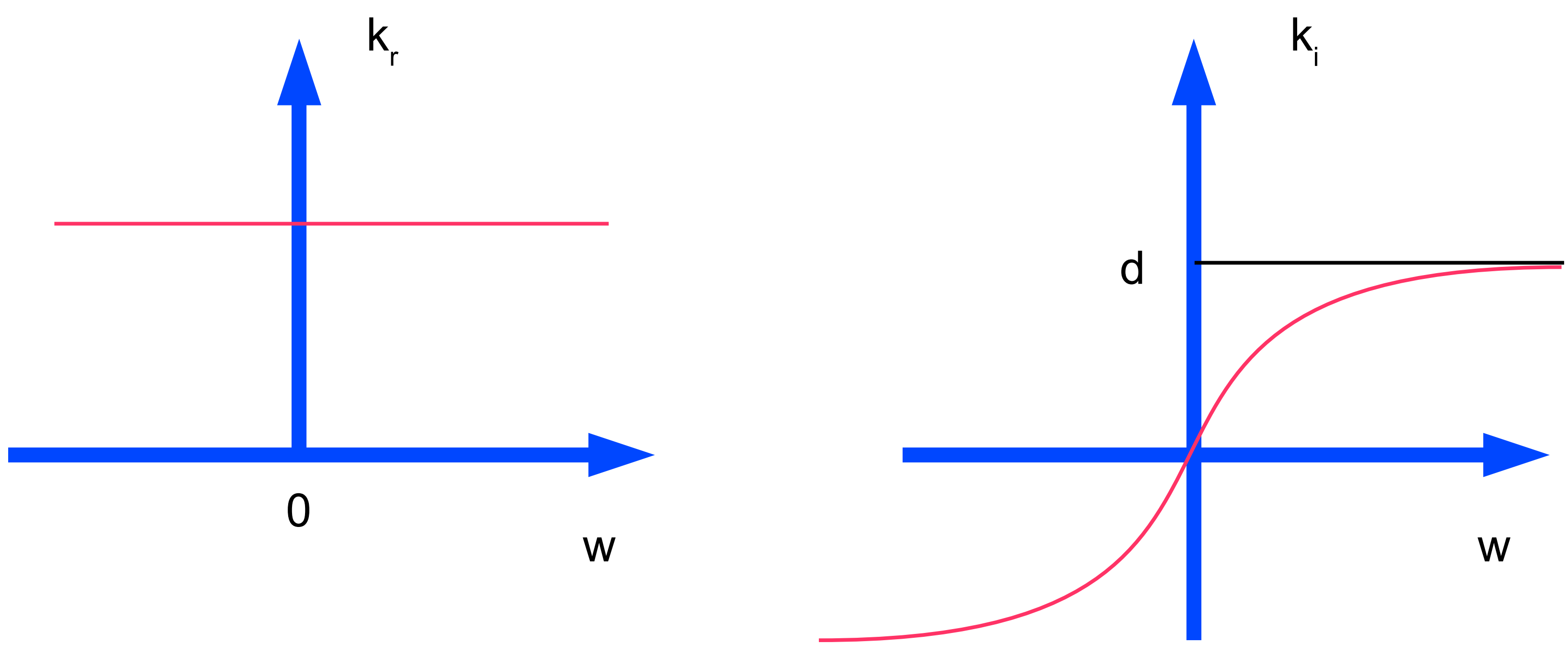


Forms of internal damping

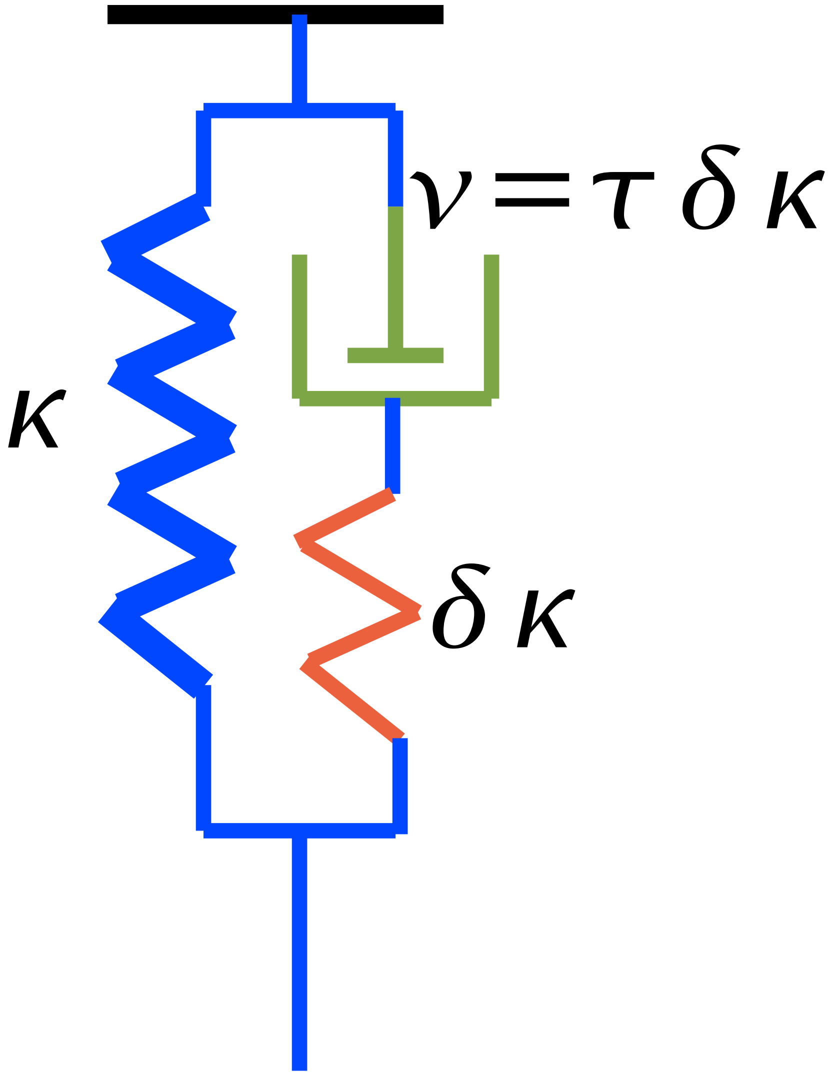
$$\kappa(1+i\,\delta(\omega))=\kappa_r(\omega)+i\,\kappa_i(\omega)$$

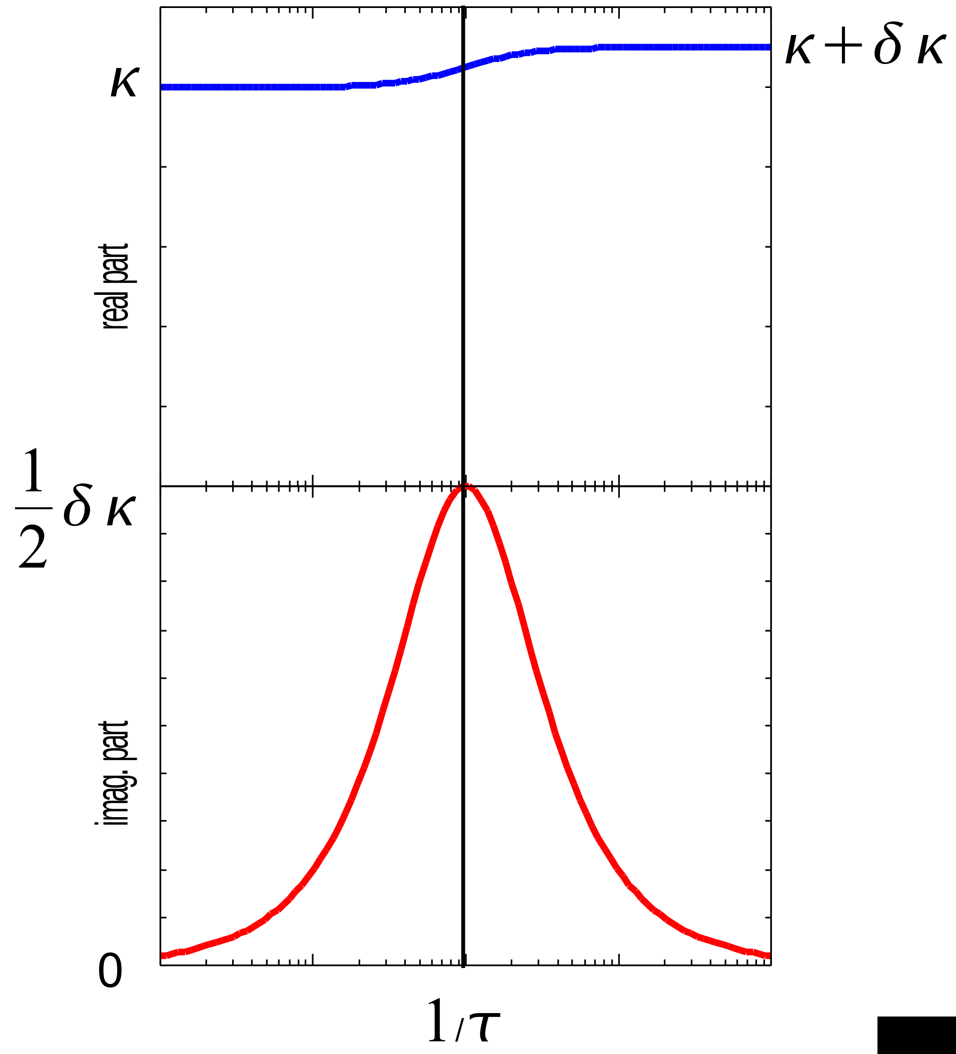
Nothing said so far about $k_i(w)$!

Kramer-Kronig relationship relates k_i to k_r

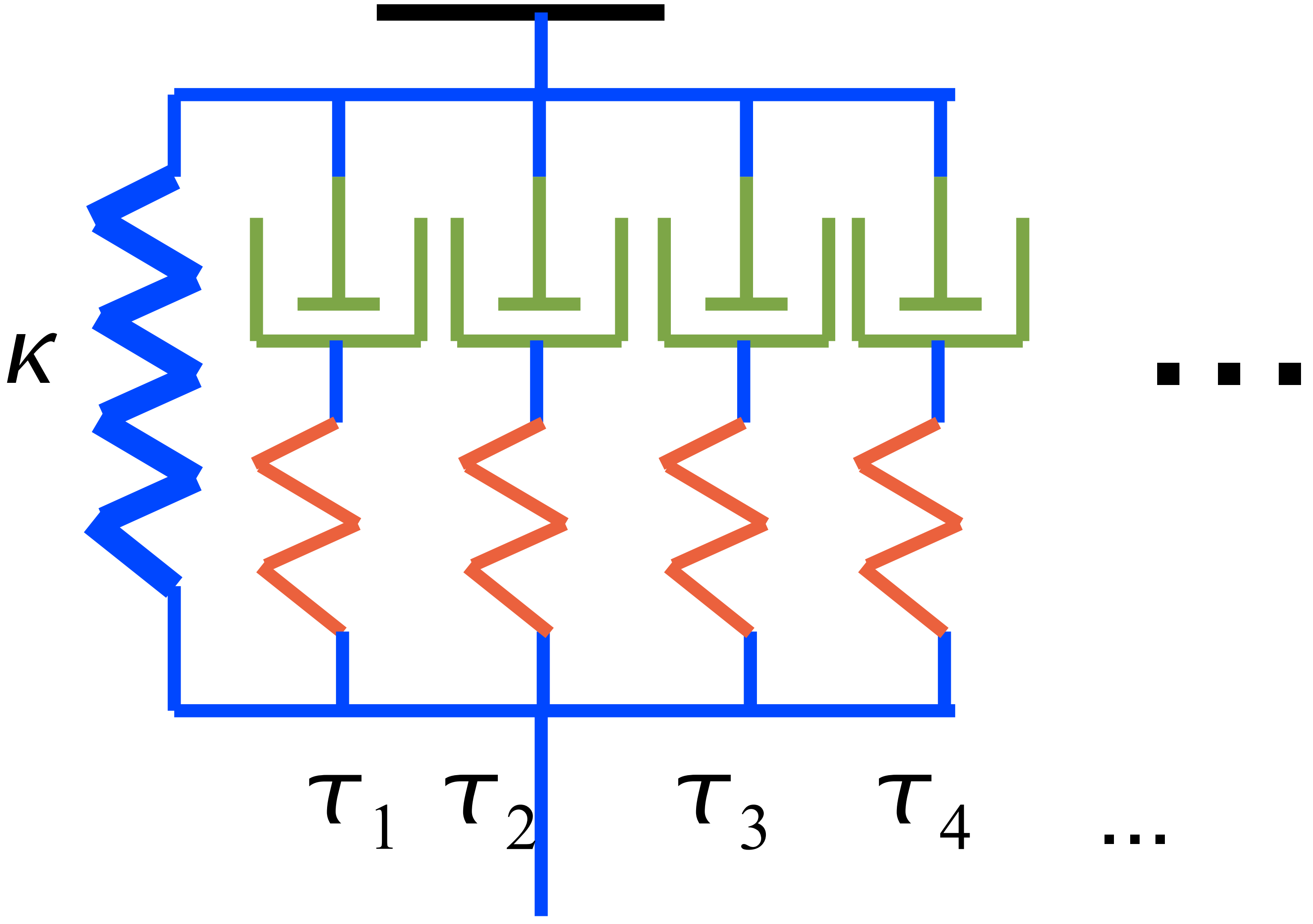


Maxwell model

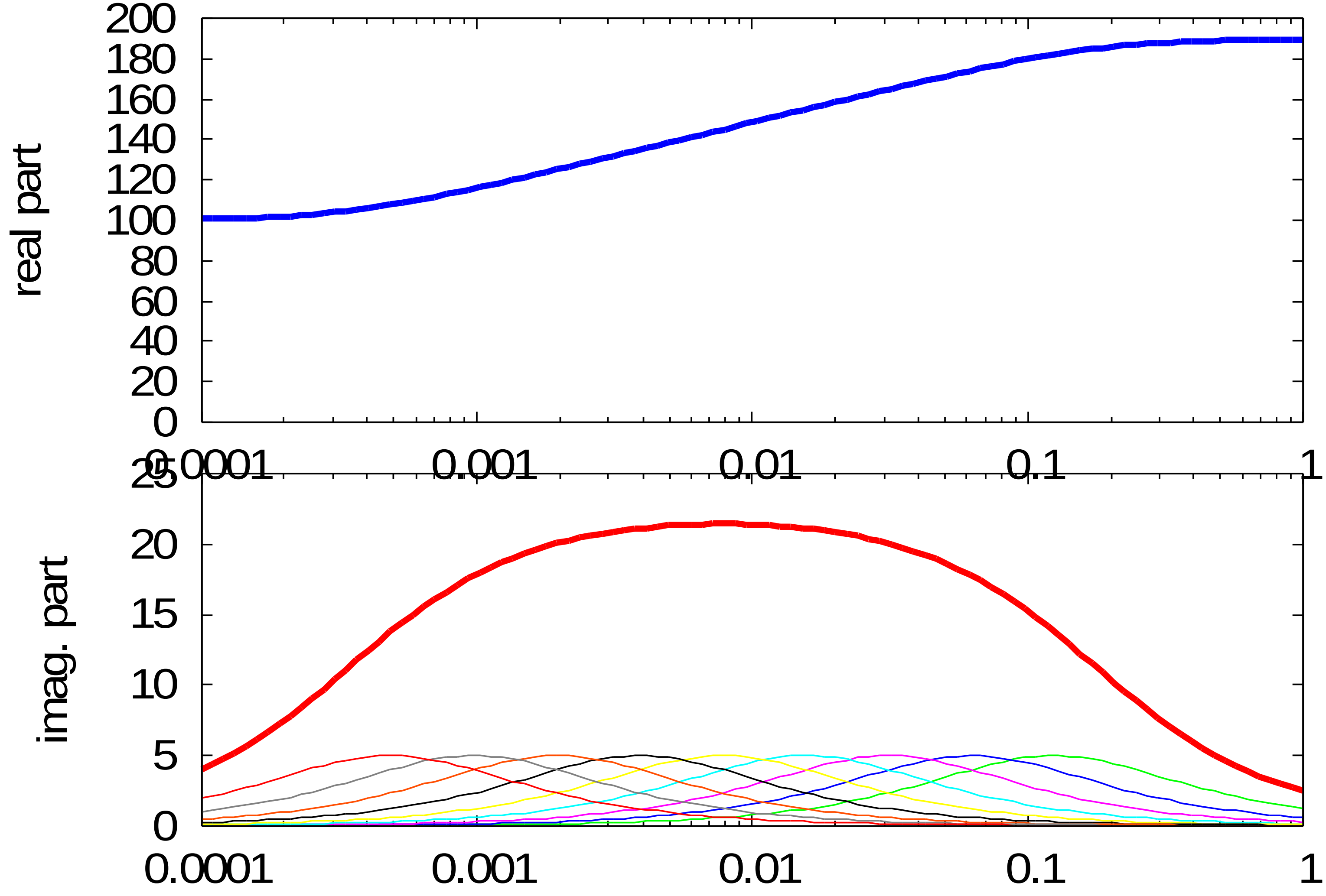




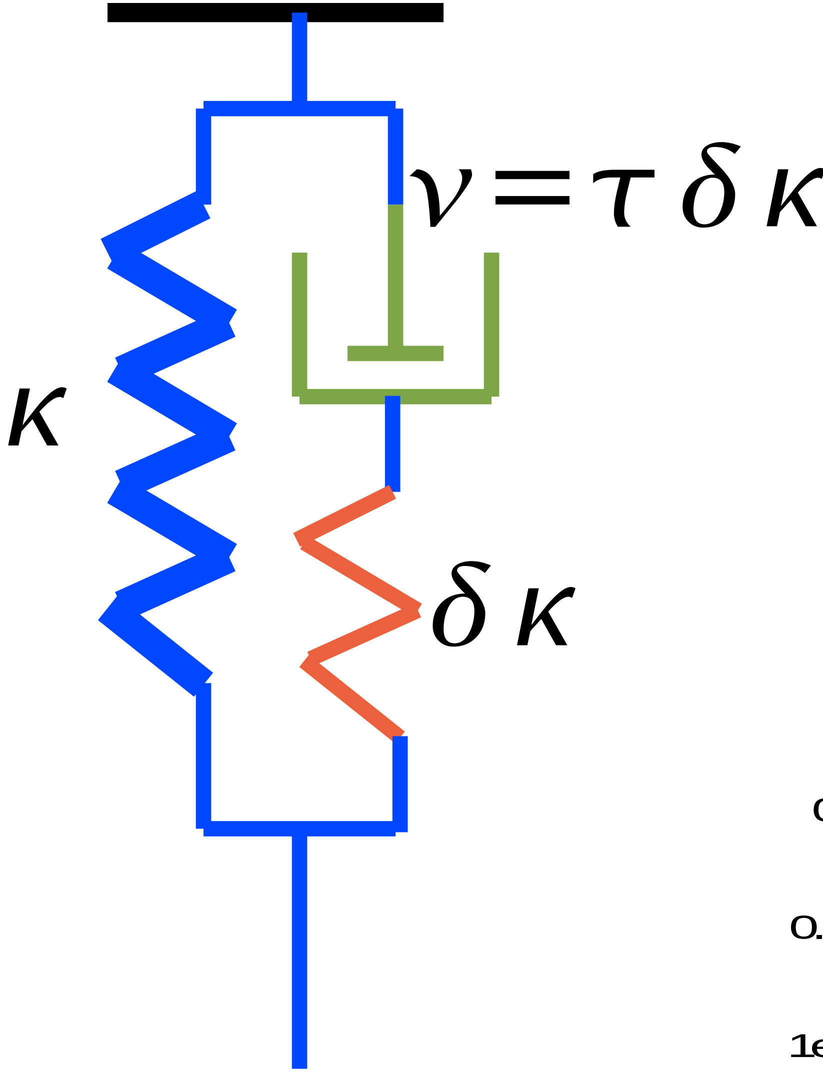
Extended Maxwell model

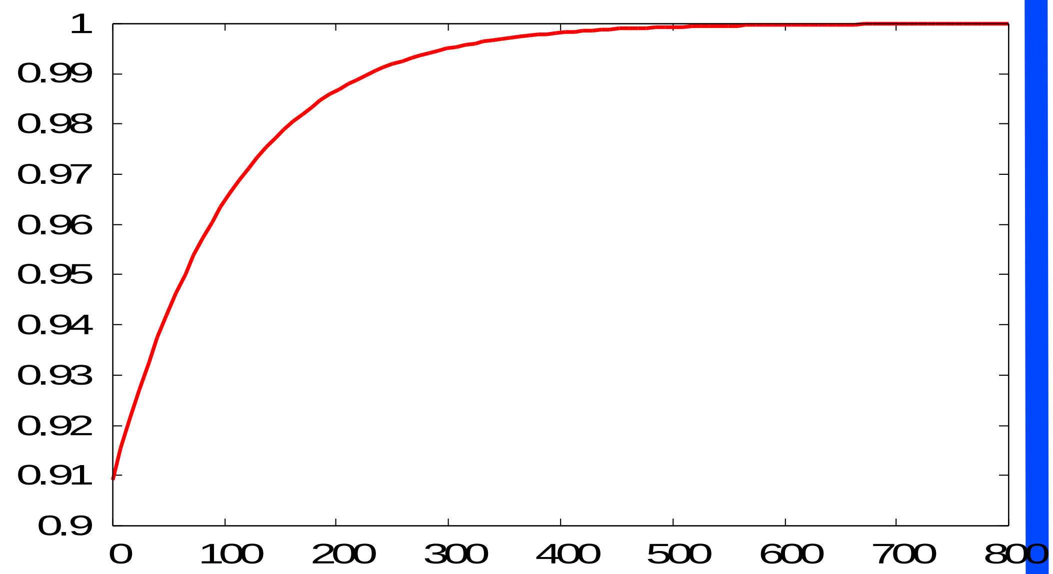


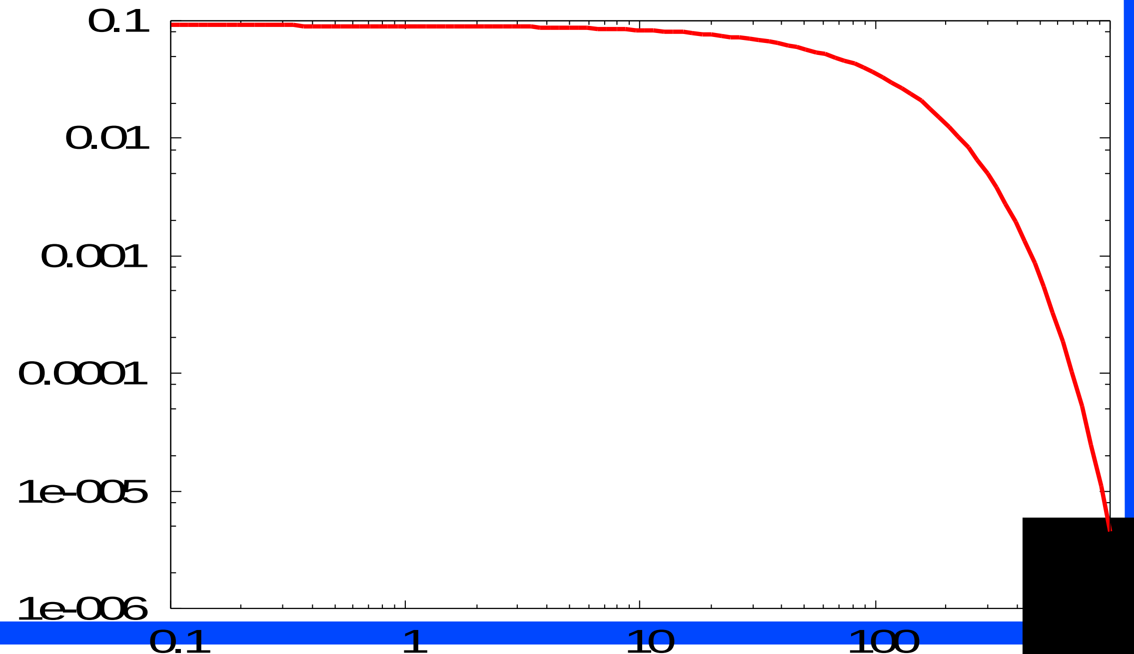
Extended Maxwell model

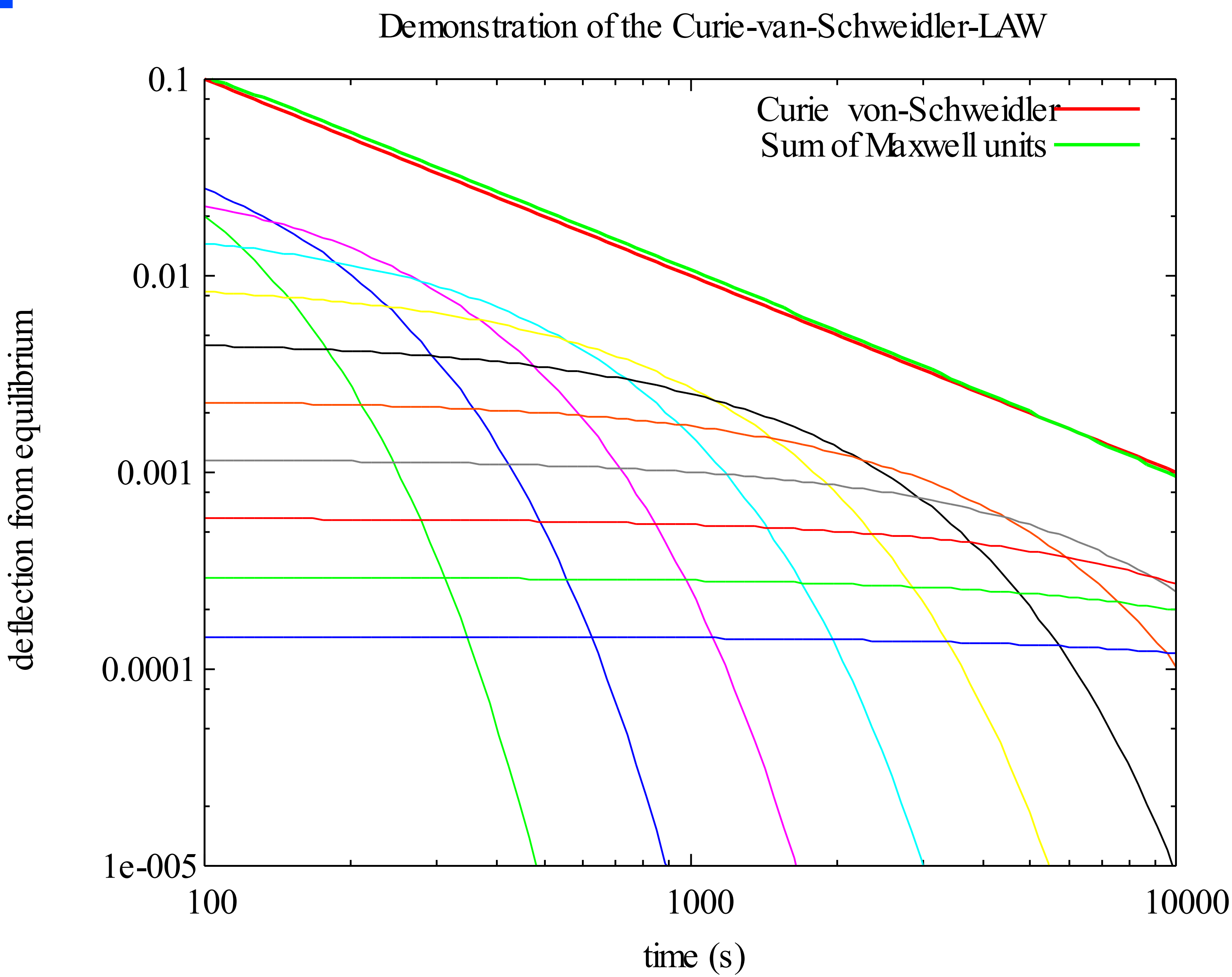


Maxwell model in the time domain

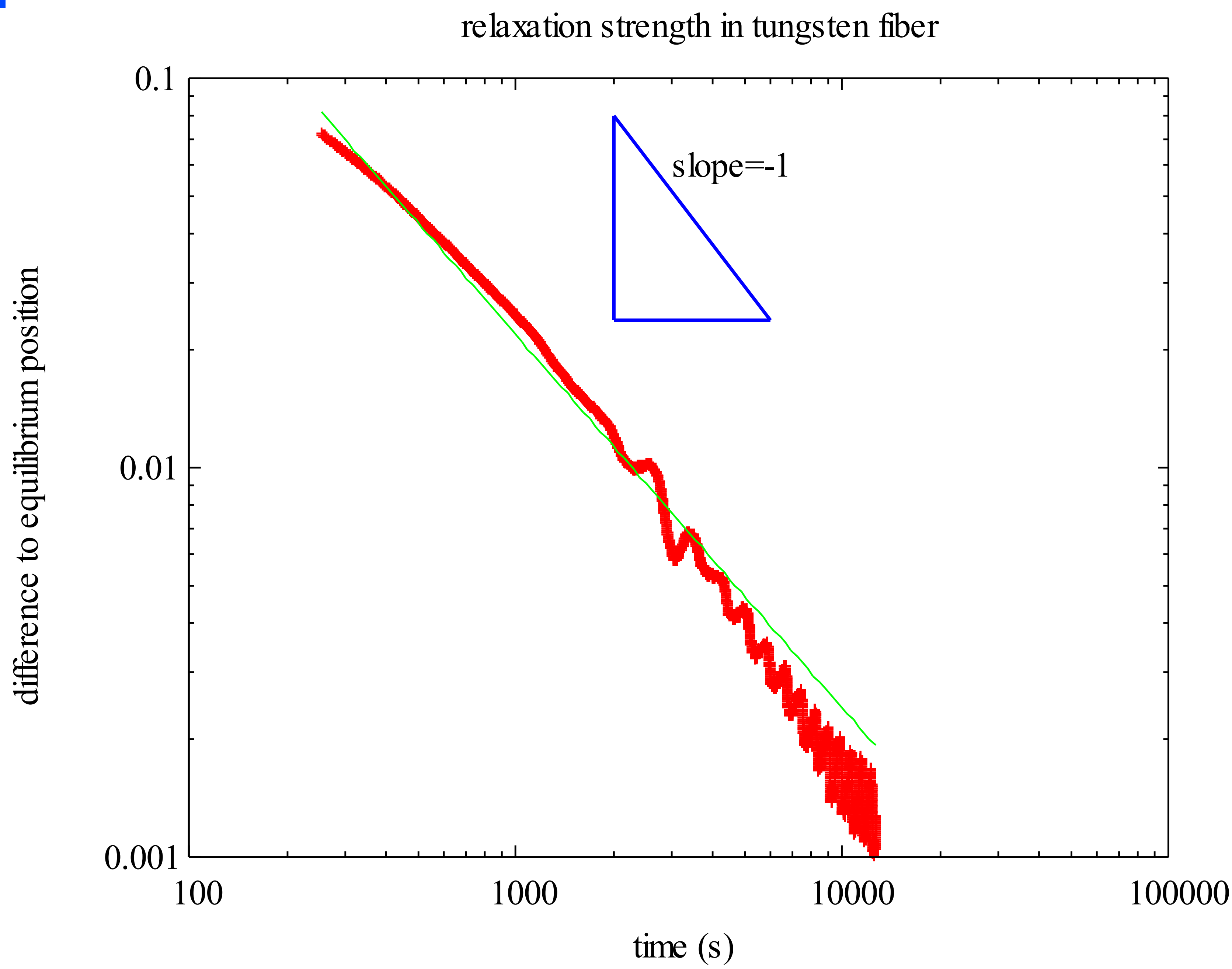








Demonstration of the Curie-van-Schweidler-LAW
Curie von-Schweidler
Sum of Maxwell units
0.1
0.01
0.001
0.0001
1e-005
100
1000
10000
time (s)
deflection from equilibrium


relaxation strength in tungsten fiber
slope=-1
0.1
0.01
0.001
100
1000
10000
100000
time (s)
difference to equilibrium position